

Refinement of the Crystal Structures of Ammonium Dihydrogen Phosphate and Ammonium Dihydrogen Arsenate

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The crystal structures of ADP, $\text{NH}_4\text{H}_2\text{PO}_4$, and ADA, $\text{NH}_4\text{H}_2\text{AsO}_4$, which crystallize in space group $I42d$ and are isomorphous with the tetragonal phase of KH_2PO_4 , have been refined at room temperature with single-crystal X-ray diffractometer data. For ADP, $a=7.4997$ (4), $c=7.5494$ (12) Å and R equals 0.021 ($R_w=0.030$) for 312 observed reflections. For ADA, $a=7.6998$ (5), $c=7.7158$ (12) Å and R equals 0.015 ($R_w=0.019$) for 585 observed reflections. The O-H...O bonds are slightly shorter in ADP than in ADA while the reverse is true for the N-H...O bonds. The difference in the N-H...O bond lengths is consistent with the difference in the thermal-expansion behavior of the two compounds.

Introduction

Arsenates and phosphates isomorphous with the tetragonal form of KH_2PO_4 (KDP) are RbH_2PO_4 (RDP), $\text{NH}_4\text{H}_2\text{PO}_4$ (ADP), KH_2AsO_4 (KDA), RbH_2AsO_4 (RDA), CsH_2AsO_4 (CDA) and $\text{NH}_4\text{H}_2\text{AsO}_4$ (ADA). This series offers an opportunity of examining a structure type with various combinations of the cationic and anionic groups. The structural investigation of the series may help in the interpretation of their physical properties, among which we are mostly interested in the thermal-expansion behavior (Khan, 1966; Deshpande & Khan, 1963, 1965*a, b*, 1966; Deshpande, Khan & Pardikar, 1968). Careful neutron-diffraction studies at room temperature have been performed on KDP (Bacon & Pease, 1953) and ADP (Tenzer, Frazer & Pepinsky, 1958). Crystal structures of ADP, RDP, KDA and ADA were refined by Khan (1966) in an attempt to understand the thermal expansion characteristics of this group of crystals but this work was of limited accuracy because of the small number of observed data collected in each case by the film method and because of the modest computational facilities available at that time. In this paper we report the results of a careful study at room temperature ($\approx 22^\circ\text{C}$) on ADP and ADA.

Ammonium dihydrogen phosphate has recently been found in bat guano in Murra-et-elevyn cave in Western Australia as a naturally occurring mineral and is called biphosphammite (Pryce, 1972).

Experimental

Commercially supplied crystalline powder of ADP was dissolved in water and recrystallized by slow evaporation. ADA was prepared by evaporating a 1:1 mixture of As_2O_5 and $(\text{NH}_4)_2\text{CO}_3$ in water. The crystals were ground to obtain spherical specimens of convenient size. Intensity data were collected on a four-circle automatic diffractometer using crystal-reflected monochro-

matic Ag $K\alpha$ radiation ($\lambda=0.5608$ Å). A list of the computer programs used in this work and a description of the procedures followed in data collection and processing is given in Baur & Khan (1970). In addition *ORTEP* (Johnson, 1965) was used. Detailed information on ADP and ADA is summarized in Table 1. Absorption corrections were applied for ADA because the minimum and maximum transmission factors differ by 3%. For ADP the difference was much less than 1% and, therefore, the absorption correction was neglected. Observed structure factors were obtained after application of the Lorentz-polarization corrections. The general equivalent positions are (0,0,0;

Table 1. $\text{NH}_4\text{H}_2\text{PO}_4$, ADP, and $\text{NH}_4\text{H}_2\text{AsO}_4$, ADA, crystal data and information regarding the data collection

	ADP	ADA
a	7.4997 (4) Å*	7.6998 (5) Å†
c	7.5494 (12)*	7.7158 (12)†
V	424.6 Å ³	457.4 Å ³
Z	4	4
F.W.	114.981	158.929
D_{calc}	1.798 g cm ⁻³	2.307 g cm ⁻³
D_{meas}	1.800 g cm ⁻³ ‡	2.313 g cm ⁻³ ‡
Space group	$I42d$	$I42d$
Diameter of the crystal	0.27 mm	0.25 mm
$\mu(\text{Ag } K\alpha)$	2.9 cm ⁻¹	40.7 cm ⁻¹
μR	0.04	0.55
Number of non-unique reflections measured	3194	5456
$R(I)$ §	0.065	0.053
Number of unique reflections	401	735
Number of $I_{hkl}=0(I < 2\sigma)$	89	150
Number of reflections used in refinement	312	585
$2\theta_{\text{max}}$	60°	60°
$\sin \theta_{\text{max}}/\lambda$	0.89 Å ⁻¹	0.89 Å ⁻¹

* Deshpande & Khan (1963).

† Deshpande & Khan (1965*b*).

‡ Averaged from Groth (1908).

§ R between equivalent reflections $\sum w| \left(\frac{\sum wI}{\sum w} \right) - I | / \sum wI$.

Table 2. $\text{NH}_4\text{H}_2\text{PO}_4$, ADP, and $\text{NH}_4\text{H}_2\text{AsO}_4$, ADA positional parameters in fractions of the cell edges and thermal parameters ($\times 10^4$) with their standard deviations

The form of the temperature factor is $\exp[-(\beta_{11}h^2 + \beta_{22}k^2 + \beta_{33}l^2 + 2\beta_{12}hk + 2\beta_{13}hl + 2\beta_{23}kl)]$ for the P, As, N and O atoms. Isotropic temperature factors for the H atoms are listed under β_{11} .

	Site symmetry	x	y	z	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
(a) ADP										
P	$\bar{4}$	0	0	0	55 (1)	55 (1)	84 (1)	0	0	0
N	$\bar{4}$	0	0	$\frac{1}{2}$	110 (2)	110 (2)	88 (3)	0	0	0
O	1	0.0843 (1)	0.1466 (1)	0.1151 (1)	75 (2)	83 (2)	118 (2)	13 (1)	-22 (2)	-37 (2)
H(n)	1	-0.002 (5)	0.089 (3)	0.563 (2)	2.8 (5) \AA^2					
H(o)	2	$\frac{1}{4}$	0.150 (6)	$\frac{1}{8}$	5 (1)					
(b) ADA										
As	$\bar{4}$	0	0	0	57 (1)	57 (1)	89 (1)	0	0	0
N	$\bar{4}$	0	0	$\frac{1}{2}$	112 (2)	112 (2)	89 (3)	0	0	0
O	1	0.0869 (2)	0.1578 (2)	0.1233 (2)	78 (2)	88 (2)	145 (2)	19 (1)	-28 (2)	-48 (2)
H(n)	1	0.014 (5)	0.120 (3)	0.567 (3)	2.4 (6) \AA^2					
H(o)	2	$\frac{1}{4}$	0.199 (9)	$\frac{1}{8}$	4.6 (2)					

$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) + (x, y, z; \bar{x}, \bar{y}, \bar{z}; \bar{y}, \bar{x}, \bar{z}; y, \bar{x}, \bar{z}; \frac{1}{2} - x, y, \frac{1}{4} - z; \frac{1}{2} + x, \bar{y}, \frac{1}{4} - z; \frac{1}{2} + y, x, \frac{1}{4} + z; \frac{1}{2} - y, \bar{x}, \frac{1}{4} + z)$. This generally adopted setting is the same as first used by West (1930) for KDP and is obtained by a rotation of 90° about the $\bar{4}$ axis relative to the setting in *International Tables for X-ray Crystallography* (1965) for the space group $\bar{4}2d$. In the least-squares refinement of the data extinction coefficients and the anisotropic temperature factors of the heavy atoms were adjusted, while the hydrogen atoms were refined with isotropic temperature factors. Anomalous dispersion corrections were made for the As atom in ADA. The initial atomic positions of the hydrogen atoms of the NH_4^+ groups, H(n), were taken as determined by Tenzer *et al.* (1958) in ADP. The hydrogen atoms, H(o), of the $(\text{H}_2\text{XO}_4)^-$ groups ($\text{X} = \text{P}^{5+}$ or As^{5+}) were refined in the position $(0, 0, 0; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) + (\frac{1}{4}, y, \frac{1}{8}; \frac{3}{4}, \bar{y}, \frac{1}{8}; \bar{y}, \frac{1}{4}, \frac{7}{8}; y, \frac{3}{4}, \frac{7}{8})$ starting with a value of y equal to y_{oxygen} . The atomic scattering factors for P, As, N, O^{-1} and H were taken from *International Tables for X-ray Crystallography* (1962) and $\Delta f'$ and $\Delta f''$ for the As atom for Ag $K\alpha$ radiation were used as given by Cromer (1965). The final parameters (Table 2) resulted in R of 0.021 ($R_w = 0.030$, extinction coefficient $C = 0.19 \times 10^{-4}$) for ADP and $R = 0.015$ ($R_w = 0.019$, $C = 0.69 \times 10^{-5}$) for ADA. The standard errors of an observation of unit weight are 0.38 for ADP and 0.47 for ADA. The maximum density in a difference map of ADA [with As, O, N and H(n) subtracted] is 0.16 e \AA^{-3} . This maximum is in a location far removed from any atomic position and is slightly larger than the density at the H(o) position. The F_o and F_c values are given in Table 3(a) and (b).

Discussion of the results

A view of the crystal structure, based on the coordinates of ADP, is shown in Fig. 1. Table 4 gives selected interatomic distances and bond angles in the two structures. It was somewhat disappointing to note that in spite of low final R values the hydrogen atom posi-

tions determined here in ADP do not compare well with those found by Tenzer, Frazer & Pepinsky (1958) in their neutron-diffraction study. In ADA the atomic parameters of the hydrogen atoms must be even more inaccurate because of the presence of the As atom. Table 4, therefore, does not include atomic distances and angles involving the hydrogen atoms. The root-mean-square displacements of atoms along the principal axes of the thermal ellipsoids and their orientations relative to the crystallographic axes are listed in Table 5. The two structures differ only in the size of the tetrahedral group XO_4^{3-} . Both the tetrahedral groups PO_4^{3-} and AsO_4^{3-} are slightly flattened into the (001) plane as

Table 3. Observed and calculated structure factors

(a) F_o , F_c and σ 's ($\times 5$) for $\text{NH}_4\text{H}_2\text{PO}_4$. Unobserved structure factors omitted.

L	FO	FC	SIG	L	FO	FC	SIG	L	FO	FC	SIG	L	FO	FC	SIG	L	FO	FC	SIG
1	10	10	10	1	10	10	10	1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10	9	10	10	10	9	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
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16	10	10	10	16	10	10	10	16	10	10	10	16	10	10	10	16	10	10	10
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18	10	10	10	18	10	10	10	18	10	10	10	18	10	10	10	18	10	10	10
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20	10	10	10	20	10	10	10	20	10	10	10	20	10	10	10	20	10	10	10
21	10	10	10	21	10	10	10	21	10	10	10	21	10	10	10	21	10	10	10
22	10	10	10	22	10	10	10	22	10	10	10	22	10	10	10	22	10	10	10
23	10	10	10	23	10	10	10	23	10	10	10	23	10	10	10	23	10	10	10
24	10	10	10	24	10	10	10	24	10	10	10	24	10	10	10	24	10	10	10
25	10	10	10	25	10	10	10	25	10	10	10	25	10	10	10	25	10	10	10
26	10	10	10	26	10	10	10	26	10	10	10	26	10	10	10	26	10	10	10
27	10	10	10	27	10	10	10	27	10	10	10	27	10	10	10	27	10	10	10
28	10	10	10	28	10	10	10	28	10	10	10	28	10	10	10	28	10	10	10
29	10	10	10	29	10	10	10	29	10	10	10	29	10	10	10	29	10	10	10
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31	10	10	10	31	10	10	10	31	10	10	10	31	10	10	10	31	10	10	10
32	10	10	10	32	10	10	10	32	10	10	10	32	10	10	10	32	10	10	10
33	10	10	10	33	10	10	10	33	10	10	10	33	10	10	10	33	10	10	10
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35	10	10	10	35	10	10	10	35	10	10	10	35	10	10	10	35	10	10	10
36	10	10	10	36	10	10	10	36	10	10	10	36	10	10	10	36	10	10	10
37	10	10	10	37	10	10	10	37	10	10	10	37	10	10	10	37	10	10	10
38	10	10	10	38	10	10	10	38	10	10	10	38	10	10	10	38	10	10	10
39	10	10	10	39	10	10	10	39	10	10	10	39	10	10	10	39	10	10	10
40	10	10	10	40	10	10	10	40	10	10	10	40	10	10	10	40	10	10	10
41	10	10	10	41	10	10	10	41	10	10	10	41	10	10	10	41	10	10	10
42	10	10	10	42	10	10	10	42	10	10	10	42	10	10	10	42	10	10	10
43	10	10	10	43	10	10	10	43	10	10	10	43	10	10	10	43	10	10	10
44	10	10	10	44	10	10	10	44	10	10	10	44	10	10	10	44	10	10	10
45	10	10	10	45	10	10	10	45	10	10	10	45	10	10	10	45	10	10	10
46	10	10	10	46	10	10	10	46	10	10	10	46	10	10	10	46	10	10	10
47	10	10	10	47	10	10	10	47	10	10	10	47	10	10	10	47	10	10	10
48	10	10	10	48	10	10	10	48	10	10	10	48	10	10	10	48	10	10	10
49	10	10	10	49	10	10	10	49	10	10	10	49	10	10	10	49	10	10	10
50	10	10	10	50	10	10	10	50	10	10	10	50	10	10	10	50	10	10	10

hydrogen atoms in the 16-fold general position and therefore it was refined in the eightfold position located on a twofold axis. Of course there can be no doubt that atom H(o) in the O-H...O bond is best described as disordered with one half of each of it residing in a general position on both sides of the eightfold special position. This is obvious from the neutron diffraction results of Tenzer *et al.* (1958) on room-temperature ADP, and of Bacon & Pease (1955) on room-temperature KDP, who both preferred the half-proton disordered model over a model in which the protons vibrate anisotropically parallel to the O-H-O vector. The actual situation must be even more complicated, as is discussed in the following paper (Baur, 1973).

Every NH_4^+ ion in the structure is coordinated by eight oxygen atoms forming two interpenetrating tetragonal disphenoids. One of the disphenoids is flat and the other steep with respect to the c axis. The N-O(f) lengths are shorter by about 0.30 Å than the N-O(s) lengths where O(f) and O(s) refer to the oxygen atoms of the flat and steep disphenoids respectively (Table 4). In KDP, however, the corresponding bond lengths from K^+ to the two sets of oxygen atoms have values which are much closer: 2.819 Å in the flat disphenoid, 2.888 Å in the steep disphenoid (Bacon & Pease, 1953). This comparison shows very clearly that in ADP (and ADA) there is a tendency for the NH_4^+ groups to engage in four hydrogen bonds within the flat disphenoid. Neutron-diffraction work on ADP by Tenzer *et al.* (1958) provides a direct evidence of the N-H...O(f) bonds. This is a specific example of the tendency of the NH_4^+ ions towards displaying smaller coordination numbers than the corresponding alkali compounds (Khan & Baur, 1972). The effect of these N-H...O bonds is reflected in the cell parameters and in the thermal expansion behavior of ADP and ADA. The c/a values for the two compounds are too large to lie on the straight line connecting the alkali isomorphs in Fig. 2. Unlike the alkali compounds the two ammonium compounds expand with temperature mainly in the ab plane. The $\bar{\alpha}_a$ values are about 20 times as large as the $\bar{\alpha}_c$ values (Table 6). As pointed out by Deshpande & Khan (1963), the N-H...O bonds which have a larger component in the ab plane tend to enforce the binding in this plane, so that the bonding in the c direction will be relatively weaker. This situa-

tion appears, therefore, to be responsible for the higher c/a values of ADP and ADA. With the increase in temperature, the initial structural constraints introduced by the presence of the N-H...O bonds are likely to decrease and the cristobalite framework of the hydrogen bonded XO_4 groups will tend to assume a structure more similar to that for the alkali isomorphs with smaller c/a values. This is accomplished by a relatively small expansion along the c axis and a much larger expansion in the ab plane as the temperature is increased. As c/a is lowered the coordination number of the ammonium ion tends towards eight, as in the alkali dihydrogen phosphates and arsenates. This is analogous to the increasing coordination numbers of NH_4^+ with rising temperature in the transition of $(\text{NH}_4)_2\text{SO}_4$ (Schlemper & Hamilton, 1966; Khan & Baur, 1972) and is an indication of the increasing alkali ion character of NH_4^+ with increasing temperature.

The average volume expansion coefficient, $\bar{\alpha}_v$, is influenced by the N-H...O bonds since the $\bar{\alpha}_v$ values for ADP and ADA are smaller than for the alkali iso-

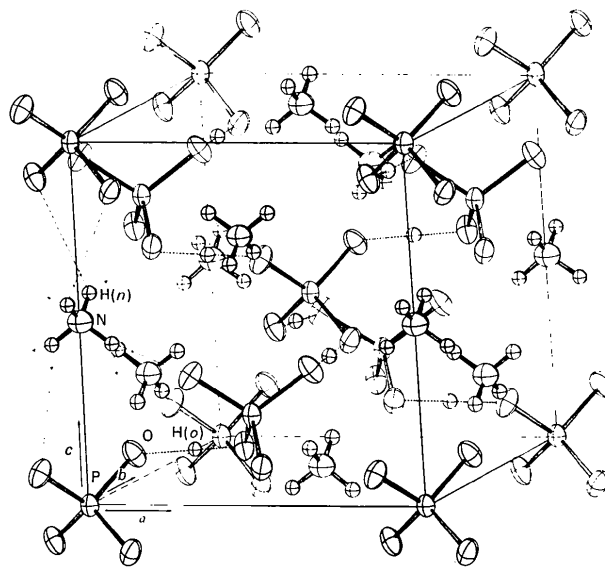


Fig. 1. A perspective view of the unit cell of $\text{NH}_4\text{H}_2\text{PO}_4$, ADP. Atoms are drawn at their 50% probability level with hydrogen atoms H(n) and H(o) included with a small arbitrary value of the isotropic temperature factor equal to 0.1 \AA^2 .

Table 6. Lattice parameters at room temperature (22°C) and average linear and volume expansion coefficients ($\bar{\alpha}_a$, $\bar{\alpha}_c$ and $\bar{\alpha}_v$) for KDP-type crystals

Crystal	$a(\text{\AA})$	$c(\text{\AA})$	$\bar{\alpha}_a (\times 10^6 \text{ }^\circ\text{C}^{-1})$	$\bar{\alpha}_c (\times 10^6 \text{ }^\circ\text{C}^{-1})$	$\alpha_v (\times 10^6 \text{ }^\circ\text{C}^{-1})$	Reference
ADP	7.4997 (4)	7.5494 (12)	39.3	1.9	80.5	Deshpande & Khan (1963)
ADA	7.6998 (5)	7.7158 (12)	22.3	1.0	45.6	Deshpande & Khan (1965b)
KDP	7.4520 (4)	6.9670 (4)	26.9	46.6	100.4	Sirdeshmukh & Deshpande (1967)
KDA	7.6290 (4)	7.1612 (9)	24.9	50.1	99.9	Deshpande & Khan (1965a)
RDP	7.6057 (9)	7.2998 (14)	27.8	52.0	107.6	Deshpande & Khan (1966)
RDA	7.7950 (7)	7.4732 (11)	20.8	46.9	88.5	Deshpande, Khan & Pardikar (1968)
CDA*	7.98 (1)	7.87 (2)				Ferrari, Nardelli & Cingi (1956)

* Room temperature not known.

morphs (Table 6). Among the two ammonium salts, the average expansion coefficient for ADA is significantly smaller than for ADP. The difference between the $\bar{\alpha}_v$ values measured by Deshpande & Khan (1963, 1965*b*) has also been confirmed in an independent study by Haussühl (1964).

The N-H...O bonds are shorter in ADA than in ADP by 0.020 Å, which is a highly significant difference. This difference is the only structural feature which can be correlated with the difference in the thermal expansion behavior of ADP and ADA. It probably is safe to assume that during the thermal expansion of the structure the $\text{PO}_4^{3-}/\text{AsO}_4^{3-}$ and NH_4^+ groups will change their individual size and shape much less than their relative positions. In other words the thermal-expansion behavior will be governed mostly by the strengths of the hydrogen bonds connecting one group to another. Since O-H...O bonds in ADA are longer (weaker) than in ADP the smaller value of the volume expansion coefficient of ADA can only be attributed to the shorter (stronger) N-H...O bonds in this structure. The O-H...O bonds are longer by 0.022 Å and the N-H...O bonds are shorter by 0.020 Å in ADA, but there are twice as many N-H...O bonds as O-H...O bonds in the structure and therefore their effect may be reflected in the smaller volume-expansion coefficient.

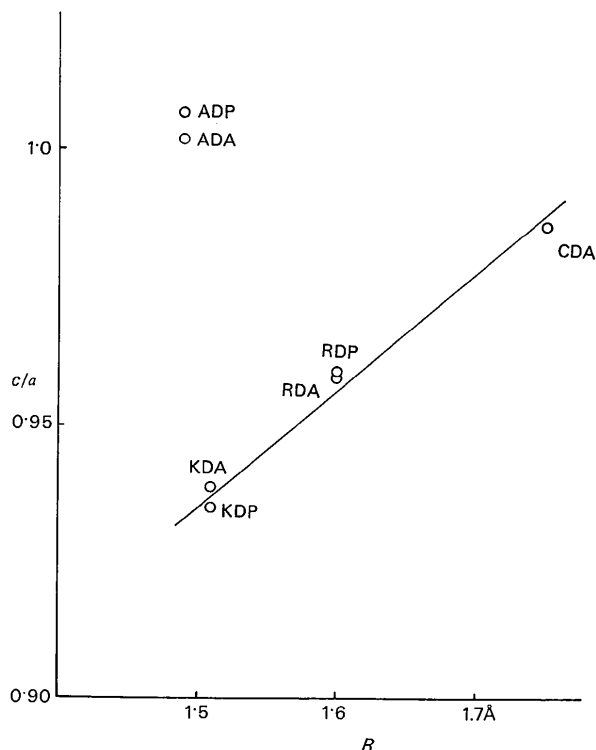


Fig. 2. The c/a ratios vs. ionic radii of K^+ , Rb^+ , Cs^+ and NH_4^+ cations in KDP-type structures. Ionic radii for the alkali atoms in eightfold coordination are from Shannon & Prewitt (1969) and for NH_4^+ ions in fourfold coordination from Khan & Baur (1972).

The differences in the lengths of the O-H...O and N-H...O bonds in the two isomorphous crystal structures can be interpreted by considering the electrostatic charge balance. The oxygen atoms which are coordinating one X^{5+} ion in the two XO_4^{3-} groups are also the acceptors of N-H...O bonds and serve as donors as well as acceptors in the O-H...O bonds. Pauling (1929) introduced the concept of the electrostatic bond strength defined as the valence of a cation divided by its coordination number. According to Pauling the sum of the bond strengths around the anions is approximately equal to their valence (with opposite sign). Recently Brown & Shannon (1973) have derived bond-strength *versus* bond-length relationships for bonds between oxygen and various cations in many crystal structures by requiring that the sums of the bond strengths around the ions be equal to their valence. The bond strengths of a P-O bond in ADP and of an As-O bond in ADA are found to be almost identical (≈ 1.24 v.u.) when calculated from the equation and the proper empirical constants given by Brown & Shannon (1973). The total electrostatic balance at the oxygen atoms is then achieved through the hydrogen bonds, which means that the sum of the bond strengths provided by the N-H...O and O-H...O bonds must be constant in ADP and ADA. Since the N-H...O bond length is longer in ADP than in ADA, the O-H...O bond length is accordingly shorter in ADP than in ADA, thus maintaining the balance of bond strengths at the oxygen atom.

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Reconstruction of Local Atomic Environments in the Disordered Hydrogen-Bonded Crystal Structures of Paraelectric Ammonium Dihydrogen Phosphate and Potassium Dihydrogen Phosphate

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In crystal structures in which one equivalent position is randomly occupied by different atomic species of different sizes the adjoining atoms must exhibit positional disorder. A special case is the statistical presence or absence of hydrogen atoms as observed in $\text{NH}_4\text{H}_2\text{PO}_4$ (ADP) and KH_2PO_4 (KDP), where the bond lengths in the phosphate group will vary locally depending on whether it is a P–O or a P–O–H [P–O(*h*)] bond. The ideal shape of an individual $\text{PO}_2(\text{OH})_2$ group is predicted on the basis of the extended electrostatic valence rule and the O–P–O on P–O dependence in orthophosphate groups: P–O 1.501 Å, P–O(*h*) 1.573 Å, O(*h*)–P–O(*h*) 105.8°, O(*h*)–P–O 109.2° and O–P–O 113.9°. This calculated $\text{PO}_2(\text{OH})_2$ group is fitted into unit cells of the same dimensions but lower symmetry than those of the paraelectric $I42d$ structures of ADP and KDP. These hypothetical local environments in C_{2v}^{10} (*I*-centered) and $P2_12_12_1$ are compatible with the observed averaged structure deduced from X-ray diffraction data for ADP. The hydrogen-atom positions as calculated for the least electrostatic energy in the local environments agree with their positions as determined from neutron-diffraction data. The local environments display interatomic distances which differ from the distances in the averaged structure individually as well as in their average values. Therefore, the bond lengths derived from the refinement of averaged disordered structures should be viewed with extreme caution. The distance between the averaged atomic positions can be different from the average of the individual distances. The situation resembles the case of the effects which thermal motion has on the estimation of bond distances. In these instances a valid correction is only possible if we have information on the nature of the thermal motion or on the type of disorder in the crystal structure. Such information is generally not available.

Introduction

In many crystal structures, and particularly in those of minerals, disorder is present. It can be evident in the random occupation of one equipoint by several atomic kinds. An example is the partial substitution of Al for Si in tetrahedral coordination in feldspars. Because the bond lengths Si–O, 1.61 Å, and Al–O, 1.75 Å (Smith & Bailey, 1963) differ by 0.14 Å the positions of the oxygen atoms must be affected by this substitution. At a given tetrahedral site occupied by Al the oxygen atoms will have a larger distance to the central cation than at a location where Si is the central atom. If the distribution of the Al and Si atoms is disordered then the X-ray data refinement will yield an averaged struc-

ture. The atomic positions of the averaged structure are the centroids of the atomic distributions in many asymmetric units. Therefore, the coordinates of the averaged positions are coordinates of *apparent* sites which most likely are not occupied in any single unit cell of the structure. In the averaged structure the atomic positions in the different unit cells superpose and cause a smearing out of the observed electron densities, which usually is not distinguishable from the effects of anisotropic thermal motion of the atoms. The thermal ellipsoid contains then the effects of both thermal motion and positional disorder. In any given asymmetric unit the local environment around an Al atom will be different from the environment in the averaged structure and from the local environment